REVIEW ARTICLE OPEN ACCESS

A Heuristic Approach for optimization of Non Linear process using Firefly Algorithm and Bacterial Foraging Algorithm

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Abstract :
A comparison study of Firefly Algorithm (FA) and Bacterial Foraging Algorithm (BFO) optimization is carried out by applying them to a Non Linear pH neutralization process. In process control engineering, the Proportional, Derivative, Integral controller tuning parameters are deciding the performance of the controller to ensure the good performance of the plant. The FA and BFO algorithms are applied to obtain the optimum values of controller parameters. The performance indicators such as servo response and regulatory response tests are carried out to evaluate the efficiency of the heuristic algorithm based controllers. The error minimization criterion such as Integral Absolute Error (IAE), Integral Square Error (ISE), Integral Time Square Error (ITSE), Integral Time Absolute Error (ITAE) and Time domain specifications – rise time, Peak Overshoot and settling time are considered for the study of the performance of the controllers. The study indicates that, FA tuned PID controller provides marginally better set point tracking, load disturbance rejection, time domain specifications and error minimization for the Non Linear pH neutralization process compared to BFO tuned PID controller.
Keywords: Firefly Algorithm (FA), Bacterial Foraging Algorithm (BFO), Optimization Technique, Non-Linear Systems.

I. INTRODUCTION

Most of the real-world optimization problems are highly nonlinear and multimodal, under various complex constraints. It is more complex and tedious to find out an optimized solution for different objectives. Sometimes, even for a single objective, optimal solutions may not exist at all. In general, finding an optimal solution or even sub-optimal solutions is not an easy task. To solve the optimization problem, efficient search or optimization algorithms are needed. There are many optimization algorithms which can be classified in many ways, depending on the focus and characteristics [1].

The heuristic algorithm is such kind of optimization technique which is widely used to obtain best possible optimum solutions for the problems. By definition “A metaheuristic is a set of algorithmic concepts (emphasis added) that can be used to define heuristic methods applicable to a wide set of different problems. In other words, a metaheuristic is a general-purpose algorithmic framework that can be applied to different optimization problems with relatively few modifications” [2]. The metaheuristic algorithm is an important part of contemporary global optimization algorithms, computational intelligence and soft computing. These algorithms are usually nature-inspired with multiple interacting agents. A subset of metaheuristics are often referred as Swarm Intelligence (SI) based algorithms and these SI-based algorithms have been developed by mimicking behaviour of birds, fish, humans and others [3].

In recent years, researchers proposed a considerable number of heuristic algorithms such as Genetic Algorithm [4], Bacterial Foraging Optimization [5] Particle Swam Optimization [6], Artificial bee colony optimization [7], Cuckoo search [8], Bat algorithm [9], Firefly algorithm [10] to obtain optimal solutions for more complex engineering optimization problems. The searching time, Dimensions of search space, convergence rate, accuracy and effectiveness are important parameters to select a suitable optimization algorithm.

The Firefly Algorithm (FA) is a heuristic algorithm, inspired by the flashing behaviour of fireflies. The firefly are produces short and rhythmic bioluminescence flashing light to act as a signal to attract other fireflies to identify its mate as well as share the information of its pray [10].

More research works have been attempted on firefly algorithm to find out solution for optimization problems. Gandomi et al. (2011) performed a study on Firefly Algorithm (FA) to solve mixed continuous/ discrete structural optimization problems [11]. Senthilnath et al. (2011) performed research work on clustering on benchmark problems using Firefly Algorithm and compared the performance of the Firefly Algorithm with the Artificial Bee Colony (ABC), Particle Swarm Optimization (PSO) [12]. A research work is carried out by Raja et al. (2013) regarding an unstable first order and second order model of the process and implemented PID controller with help of firefly algorithm based optimization [13]. The study has shown the better performance of the firefly algorithm compared with the other optimization techniques. Recently, the FA is adapted to estimate and control a class of chemical process models [14].

Passino (2002) has proposed the Bacterial Foraging Algorithm based adaptive controller for a liquid level control problem. The perception of foraging activities of Escherichia coli (E. coli) bacteria is used for the optimization technique to find out the best fitted PID controller parameters by a set of artificial bacteria in the “D” dimensional search space. Many attempts by researchers have been carried out to find the optimal controller parameters using Bacterial Foraging Algorithm for different categories of engineering optimization problems . Datta et al. (2008) have proposed an improved adaptive approach involving Bacterial Foraging Algorithm to optimize both the amplitude and phase of the weights of a linear array of antennas for maximum array factor at any desired direction and nulls in specific directions[15]. In their work, it was found that Bacteria Foraging Algorithm is capable of improving the speed of convergence as well as the precision in the desired result. Bhushan et al. (2011) have been implemented the bacterial foraging algorithm for identification and high performance speed control system for a DC motor [16]. Recently, Rajinikanth and Latha (2012) discussed about the BFO-tuned I-PD controller performance on a class of time delayed unstable process models [17].

In the proposed work, Nonlinear pH neutralization process is chosen as an application for which Firefly algorithm and Bacterial Foraging Algorithm optimization technique have used to find out the best optimized PID controller parameters such as K_p,K_i K_d. The predicted controller parameters are tested in simulated environment to control the pH neutralization process and their results have been compared. The simulation results exhibited that FA based controller has an improved performance indices against BFO based controller.

The remaining part of the paper is organized as follows: Section 2 provides Firefly Algorithm and section 3 presents the concept of BFO algorithm. The section 4 brief about the pH Neutralization Process, PID Controller Structure and Controller Tuning By Heuristic Algorithms. The section 5 deals about results and discussions and followed by the conclusion of the research work in Section 6.

FIREFLY ALGORITHM (FA)

A chemically produced light is generated by fireflies at their lower Abdomen. The induced light pattern is used to establish communication with neighbour firefly to share the information about its food and also for mate. The firefly algorithm use the following three idealized rules [10, 18 and 22]

- All the fireflies are unisex so that one firefly is attracted by other fireflies regardless of their sex.
- The attractive signals of fireflies are proportional to its brightness of the light. Both attractiveness and brightness are reducing when the distance between the fireflies are increasing. Also, less bright firefly move towards another firefly which induces more luminance. In case, all fireflies have lesser luminance, they move randomly till identify the brighter firefly.
- The brightness of a firefly is related with the analytical form of the objective function and it is assigned to guide the search process.

For a maximization problem, brightness of a firefly is considered as to be proportional to the value of cost function.

Fundamentals of the FA

The most important parameters which decide the efficiency of the FA are the variation of light intensity and attractiveness between neighbouring fireflies. Both the parameters are affected when the fireflies maintain more distance between each of them.

The equation (1) expresses the variation of brightness in the Gaussian form,

$$I(r) = I_0 e^{-\gamma r^2} \quad (1)$$

Where I = New light intensity, I_0 = Original light intensity, γ = light absorption coefficient and r = Distance between fireflies.

Firefly’s attractiveness is proportional to the light intensity of the nearby firefly. The attractiveness β of firefly can be given by

$$\beta = \beta_0 e^{-\gamma r^2} \quad (2)$$

Where, β = attractiveness coefficient, and β0 = attractiveness at r = 0.

The equation (2) can be approximated into a simple exponential format to ensure easy analysis and faster calculations.

$$\beta = \frac{\beta_0}{1+\gamma r^2} \qquad (3)$$

The equations (2) describe a characteristic distance Γ = 1/*γ* over which the attractiveness significantly changes from β_0 to $\beta_0 e^{-1}$. The attractiveness function *β(r)* can be any monotonically decreasing functions and it is given by

$$\beta(d) = \beta_0 e^{-\gamma r^m} \quad \text{Where } m \geq 1 \qquad (4)$$

For a fixed γ, the characteristic length becomes

$\Gamma=\gamma^{-1/m} \rightarrow 1,\ m \rightarrow \infty$

Conversely, for a particular length scale Γ, in an optimization problem, the parameter γ can be used as a typical initial value. This value is

$$\gamma = 1/\Gamma\, m \qquad (5)$$

The Cartesian distance between two fireflies *i* and *j* at x_i and x_j, in the *n* dimensional search space can be mathematically expresses as

(6)

$$r_{i,j} = \left\| x_i - x_j \right\| = \sqrt{\sum_{k=1}^{n} (x_{i,k} - x_{j,k})^2}$$

The movement of firefly i is attracted by another brighter firefly j is given by

$$x_{inew} = x_i + \beta_0 e^{-\gamma r_{i,j}^2}(x_j - x_i) + \psi \qquad (7)$$

Where, x_{inew} is updated (present) position of firefly, x_i is initial position of firefly and $\beta_0 e^{-\gamma r_{i,j}^2}(x_j - x_i)$ is attraction between fireflies.

Also the parameter $\psi = \alpha \notin_i$. Where, $\notin_i$ is vector of random number which is drawn from a Gaussian distribution and α is the randomization parameter.

The equation 7 implies that the updated position of the i^{th} firefly depends on initial position of the firefly, attractiveness of firefly towards the brightness and the randomization parameter.

In this study, the the firefly algorithm is assigned with the following values to obtain controller parameters. Number of fireflies (n) = 12, β_0 = 1, γ = 6 α_0 = 0.5 (gradually reduced to 0.1 in steps of 0.001 as iterations proceed) and the total number of run is chosen as 1,000.

BACTERIAL FORAGING OPTIMIZATION ALGORITHM

Bacterial Foraging Optimization (BFO) algorithm is a new division of biologically inspired computing technique introduced by Passino in 2000 . It is based on mimicking the foraging methods for positioning, handling and ingesting food behaviour of Escherichia coli (E. coli) bacteria living in human intestine [5,21]. The algorithm has an advantage of high computational efficiency, simple design procedure, and stable convergence. The flow chart shown in Fig. 1 gives the flow of BFO algorithm and its basic operations with key process.

Chemo-taxis: This process simulates the movement of an E.coli cell towards the food source with swimming and tumbling action via flagella. The bacteria can move in a particular path by swimming and can modify the direction of search during tumbling action. These two modes of operations are endlessly executed by a bacteria its whole lifetime to reach the sufficient amount of positive nutrient gradient.

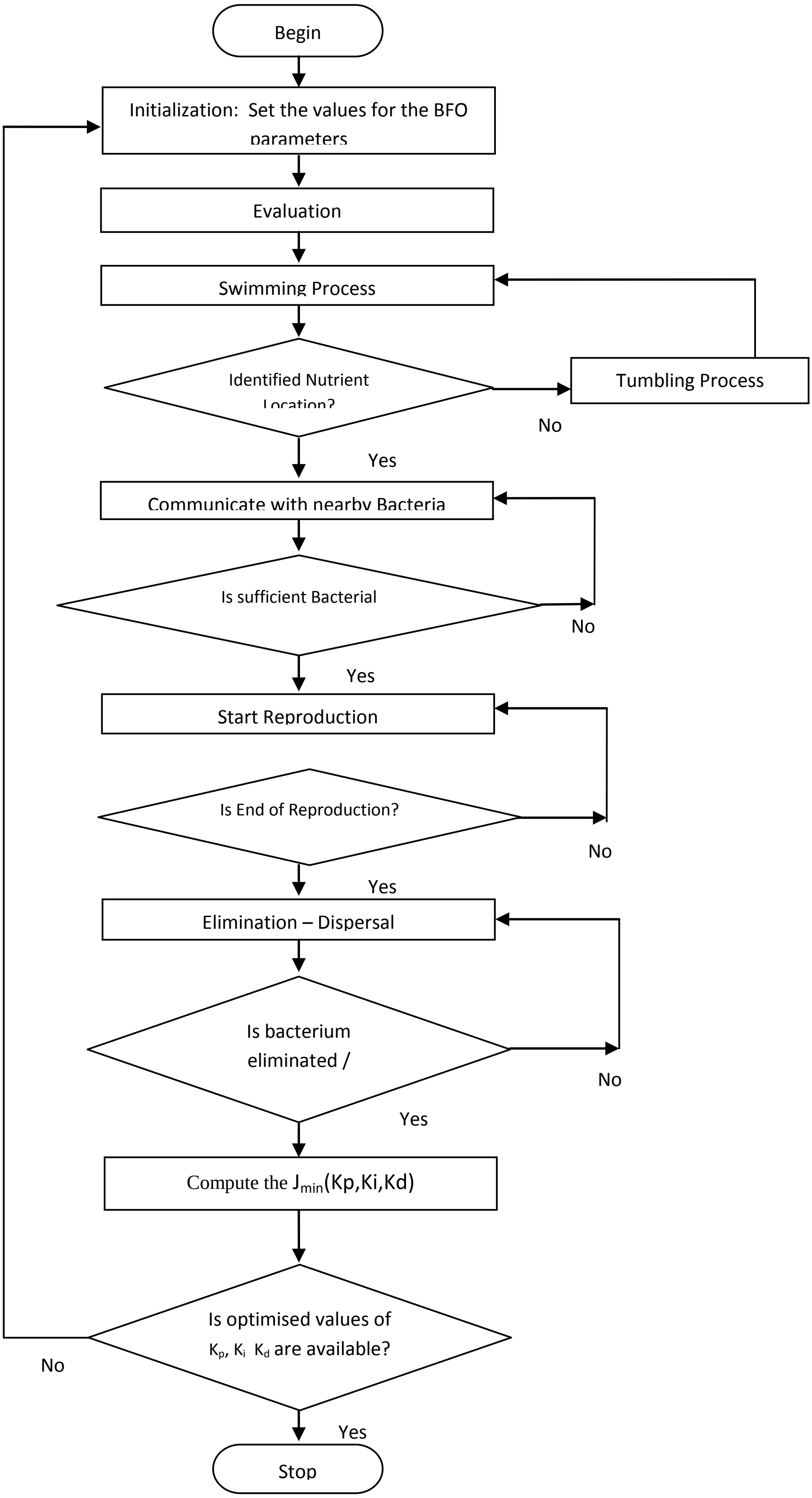

Fig. 1: Flow chart for bacterial foraging algorithm

Swarming: This process is carried out by the bacteria to acknowledge the information about optimum path of the food source with other bacteria. An attraction signal is produced for this communication between the cells in the E-coli bacteria. Another repellent signal is also produced for noxious reserve. This process helps them to increase the bacterial density at the identified food position in the chemotaxis. The attraction signal is represented by the below equation (8).

$$J_{cc}(\theta(i,j,k,l)) = \sum_{i=1}^{s} J_{cc}(\theta, \theta^{i}(j,k,l)) = X + Y$$

Where

$$X = \sum_{i=1}^{s}\left[-d_{att}\exp\left(-w_{att}\sum_{m=1}^{n}(\theta_m - \theta_m^i)^2\right)\right]$$

and

(8)

$$Y = \sum_{i=1}^{s}\left[-h_{rep}\exp\left(-w_{rep}\sum_{m=1}^{n}(\theta_m - \theta_m^i)^2\right)\right]$$

Where "s" = Total number of bacterium, "n"= Total parameters to be optimized, d_{att} = Depth of attractant signal released by a bacteria, "W_{att}" = Width of attractant signal, "h_{rep}" = height of repellent signals between bacterium, "W_{rep}" = weight of repellent signals between bacterium and $J_{cc}(\theta,(i,j,k,l))$ is the objective function value. "θ" is the point in the n dimensional search domain till the j^{th} chemotactic, k^{th} reproduction and l^{th} elimination. Also "θ_m" is the m^{th} parameter of global optimum bacteria

Reproduction: In swarming process, the bacteria are gathered as groups in the positive nutrient gradient and which may increase the bacterial density. Later, they are arranged in descending order based on its health values. The least healthy bacteria eventually expire while healthier bacteria asexually split into two bacteria and maintain the predefined population.

Elimination-Dispersal: This is the closing phase in the bacterial search. The bacterium population may decrease either gradually or suddenly depend on the environmental criteria such as change in temperature, and availability of food etc. Significant local rise of temperature may kill a group of bacteria that are currently in a region with a high concentration of nutrient gradients. Actions may take place in such a way that all the bacteria in a location are killed and eliminated (local optima) or a group is relocated (dispersed) into a new food source. The dispersal possibly compresses the chemo-taxis advancement. After dispersal, some bacteria may be located near the superior nutrient and this process is called "Migration". The above events are continued until the entire dimensional search converges to optimal solutions or total number of iterations is reached

In this study the following parameters are assigned to BFO as the preliminary process for optimization search. Number of E. coli bacteria as is ten; number of reproduction steps is assigned as four; length of a swim considered as four ; number of chemo tactic steps is selected as five; number of elimination-dispersal events are considered as two; number of bacterial reproduction is set as five, probability for bacteria eliminated /dispersed is considered as '0.25'; d_{att} is assigned as zero ; W_{att} is set as '0.5' h_{rep} is considered as '0.6' and W_{rep} is assigned as '0.6'.

APPLICATION EXAMPLE OF NON LINEAR PROCESS OPTIMIZATION USING FA AND BFO: pH NEUTRALIZATION PROCESS

The mathematical model of closed loop performance of the pH neutralization [19] is considered in this study to evaluate the performance of FA and BFO algorithms.

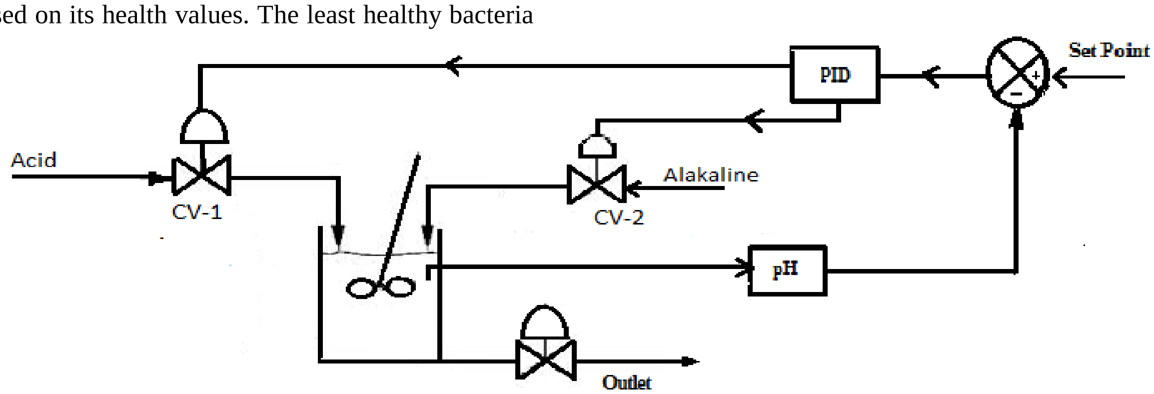

Fig. 2: pH neutralization system

Fig. 2 shows the basic concept of controlling pH neutralization process. In this, pH is controlled by maintaining appropriate flow rate of acid and/or alkaline. The mathematical model of the pH neutralization process is expressed in the form of First Order Plus Time Delay (FOPTD) structure and it is given in the equation (9)

$$G(s) = \frac{7.0921\, e^{-1.71s}}{8.54s + 1} \qquad (9)$$

PID CONTROLLER STRUCTURE

Proportional-Integral-Derivative (PID) controllers are used in the most of the process industries such as food, chemicals, pharmaceuticals, petroleum, paper industries. The PID controller provides better steady state, transient response as well as it maintain stability, smooth reference tracking and load disturbance rejection to the process [23,24]. In a closed loop control system, the controller output modify the final control element until the difference between reference input and the process output is zero irrespective of the internal and/or external disturbance signal.

The Fig. 3 shows the basic block diagram of closed loop control systems. In this, $G_c(s)$ is the PID controller and it act as the controller to control the process of $G_p(s)$. Also the R(s) is reference signal; Y(s) is controlled output signal; E(s) is error signal; $U_c(s)$ is controller output; The Fig. 4 shows the structure of parallel PID controller. The mathematical models of parallel PID controller described in the equations (10) and (11) are widely used and considered for this study.

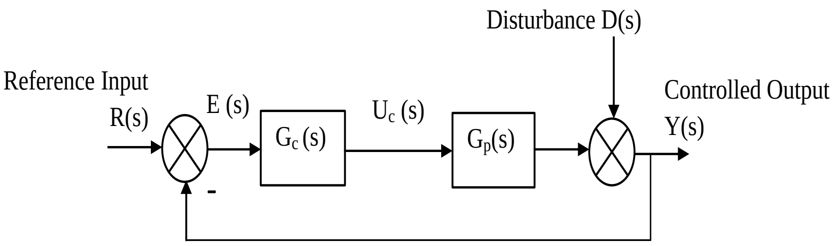

Fig. 3: Block diagram of Closed Loop Control Systems

$$G_C(s) = \left(K_p + \frac{K_i}{s} + \frac{K_d\, s}{T_f\, s + 1} \right) \qquad (10)$$

Where, $T_f = T_d/N$; T_f = Filter time constant. T_d = Derivative controller time constant (K_d/K_p); N = derivative filter constant. In this study "N" is selected as 10.

The controller output is given as

$$U_C(s) = K_p\, e(t) + K_i \int_0^T e(t)\, dt + \left(\frac{K_d}{T_f s + 1} \right) \frac{de(t)}{dt} \qquad (11)$$

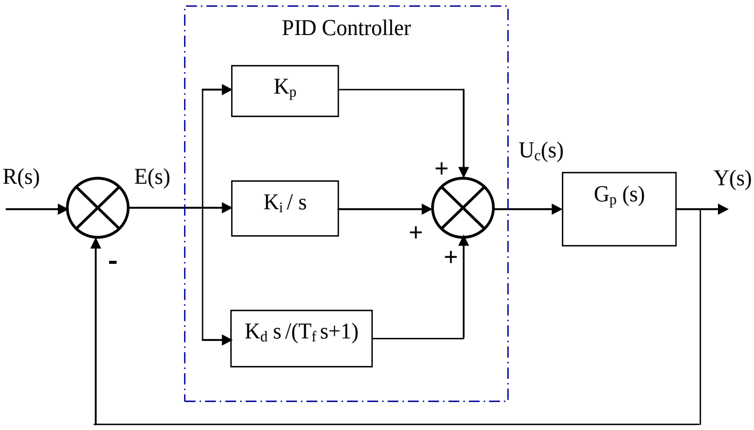

Fig. 4: Block diagram of Parallel PID control systems

CONTROLLER TUNING BY HEURISTIC ALGORITHM

The controller parameters K_p , K_i, K_d are used to determine the performance of PID controller in most of the processes. In this study, the optimum controller parameters for the PID controller are obtained from the Heuristic algorithms such as FA and BFO for the pH neutralization process. The Fig. 5 shows the basic structure of FA/BFO algorithm based PID controller tuning controller tuning.

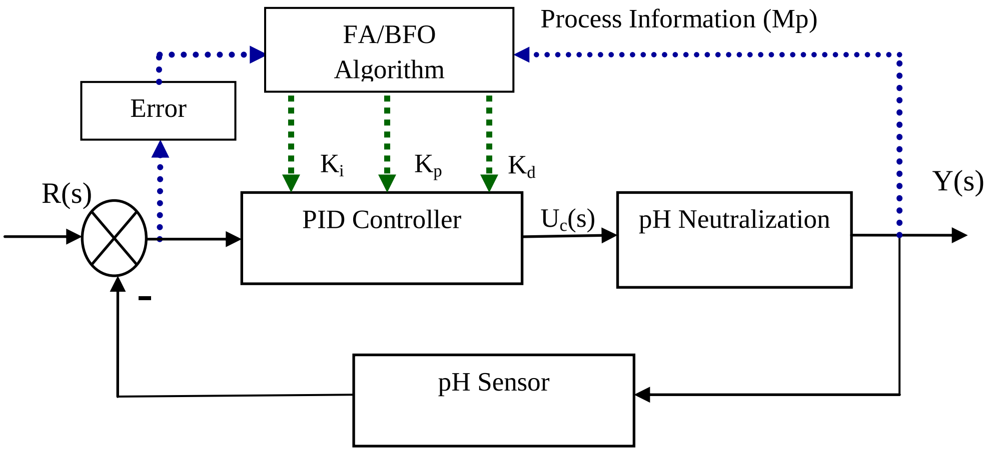

Fig. 5: Block diagram of FA/BFO based controller

Multiple Objective Performance Index (MOPI)

Initially, the boundary values of PID are assigned to guide the optimization algorithm and to attain the good accuracy. Many researchers have proposed the Multiple Objective Performance Index (MOPI) such as overshoot (M_p), settling time (t_s), steady state error (e_{ss}), rise time (t_r), gain margin (GM) and phase margin (PM) for PID controller optimization [17,20]. The following MOPI equation is considered to obtain the controller Parameter K_p, K_i and K_d for the study of heuristic algorithms performance.

$$J_{\min}(k_p, k_i, k_d) = (w_1.ISE) + (w_2.IAE) + (w_3.M_p) + (w_4.t_s) + (w_5.t_r) \quad (12)$$

Where J_{min} (K_p,K_i,K_d) - Performance criterion
ISE - Integral Square Error
IAE - Integral absolute Error
M_p - Peak Overshoot is the difference between maximum peak value of the response curve $c(t_p)$ and final value of c(t)
t_s - Settling time is time required for the response curve to reach and stay within 2% of the final value.
t_r - Rise time is time required for the response to rise from 0% to 100% of its final value.
w_1, w_2, w_3, w_4 and w_5 are weighting functions of the MOPI parameters and the value of "w" varies from 0 to 10.

The following parameters are assigned to MOPI as the preliminary process for optimization search.

- Dimension of the search is assigned as three (K_p, K_i, K_d);
- The limits of the three dimensional search space is as
  - K_p = 0% < K_p < +50%
  - K_i = 0% < K_i < +25%
  - K_d = 0% < k_d < +50%
- The weighting function values are assigned as $w_1 = w_2 = w_3 = 10$, $w_4 = w_5 = 6$.
- The reference input signal 'R(s)' is unity.
- The "t_r" is chosen as <25% of the maximum simulation time. The settling time 't_s' is selected as <50% of the maximum simulation time.
- The overshoot in the process output 'M_p' is considered as <10% of the reference signal.
- The steady state error (e_{ss}) of process output is assigned as zero.
- Maximum simulation time is 100 sec. The simulation time is selected based on the process time delay.

RESULTS AND DISCUSSIONS

Five trials are carried out for each algorithm optimization search. The convergence of the Firefly Algorithm (FA) towards finding the optimum controller parameters are presented in the Table 1. The Fig. 6

depicts the qualitative comparison of the servo response for the trial values presented in Table 1. It is interpreted that the trial value with iteration 34 is best optimal value compared to other trial values.

Table 1: Controller parameters and their performance for 5 trials of FA

Trials	Iteration	Controller Parameter			Error Minimization				Time Domain Specification		
		K_p	K_i	K_d	IAE	ISE	ITAE	ITSE	M_p %	T_r (Sec.)	T_s (Sec.)
1	35	0.3926	0.0224	0.0837	5.936	3.028	59.33	7.408	-	-	-
2	38	0.6011	0.0542	0.0391	4.238	2.679	18.31	4.7	29.2	4.83	22
3	34	0.4288	0.0491	0.0463	3.691	2.746	9.546	4.184	13	5.92	14
4	41	0.5593	0.0835	0.0365	4.449	2.859	17.33	5.647	38.9	4.85	17
5	42	0.6102	0.0921	0.0733	4.796	2.967	21.22	6.532	44.7	4.65	24

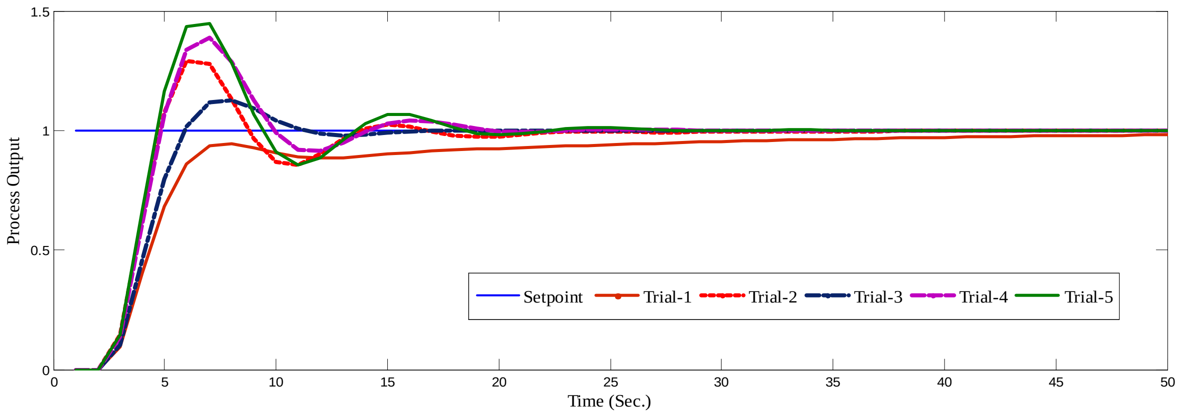

Fig 6. Servo response of PID Controller for 5 different trials of FA algorithm.

Table 2: Controller parameters and their performance for 5 trials of BFO

Trials	Iteration	Controller Parameter			Error Minimization				Time Domain Specification		
		K_p	K_i	K_d	IAE	ISE	ITAE	ITSE	M_p %	T_r (Sec.)	T_s (Sec.)
1	224	0.5925	0.0105	-0.1306	8.243	3.173	137.9	18.42	13.5	6.0	-
2	272	0.6478	0.0929	0.5311	4.693	2.919	20.19	6.252	46.0	4.55	20
3	284	0.5523	0.0438	0.5863	4.024	2.595	19.61	4.002	17.0	5.15	22
4	216	0.5826	0.0424	0.6200	4.201	2.591	23.28	4.176	19.7	4.95	24
5	256	0.6515	0.0315	0.6470	5.155	2.652	46.42	5.646	24.8	4.76	43

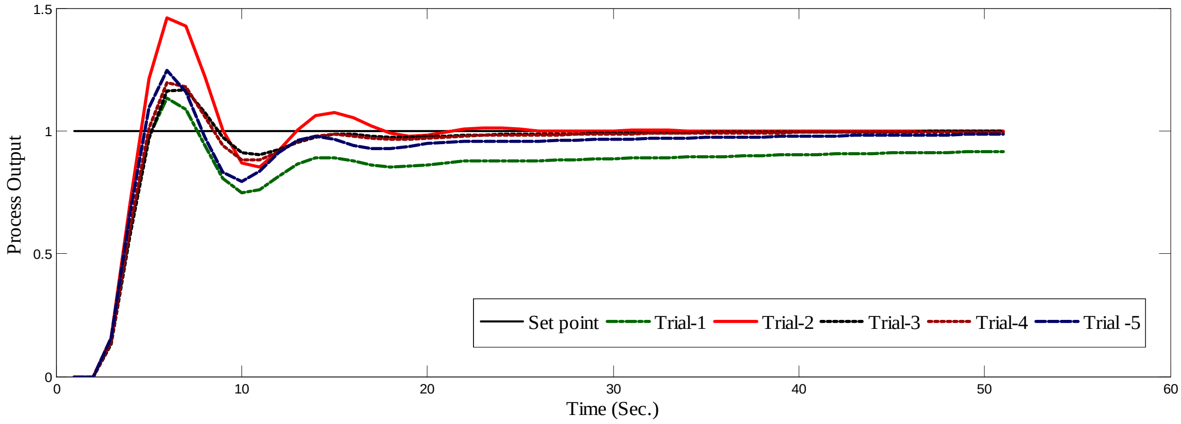

Fig 7. Servo response of PID Controller for 5 different trials of BFO algorithm.

Similar Procedure is attempted for the FOPTD model of the neutralization process through BFO algorithms. For each algorithm, five trials are executed and the obtained controller parameters are presented in the Table 2 and it is depicted in the Fig.7. From the observation it is found that the third trial with iteration 284 is best optimal controller tuning parameters compared with the remaining trial values. The best performance trial values are highlighted in the table for easy observation.

The best performance trial values produced by both the algorithm such as FA and BFO are tabulated in the table 3.

Table 3: Comparison of FA and BFO Algorithm based PID Controller performance Indices

Algorithm	**Iteration**	**Controller Parameter**			**Error Minimization**				**Time Domain Specification**		
		K_p	**K_i**	**K_d**	**IAE**	**ISE**	**ITAE**	**ITSE**	**M_P %**	**T_r (Sec.)**	**T_s (Sec.)**
FA	34	0.4288	0.0491	0.0463	3.691	2.746	9.546	4.184	13	5.92	14
BFO	284	0.5523	0.0438	0.5863	4.024	2.595	19.61	4.002	17.0	5.15	22

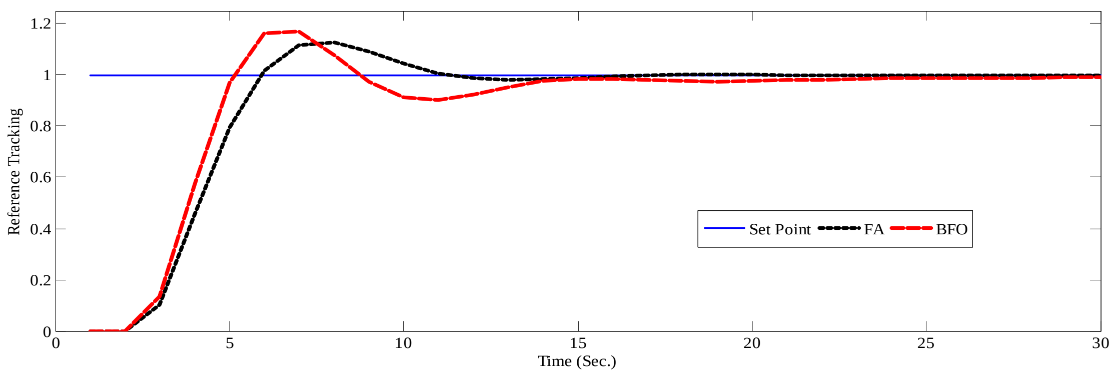

Fig.8: Comparison of Servo Response of FA and BFO algorithm based PID Controllers Tuning

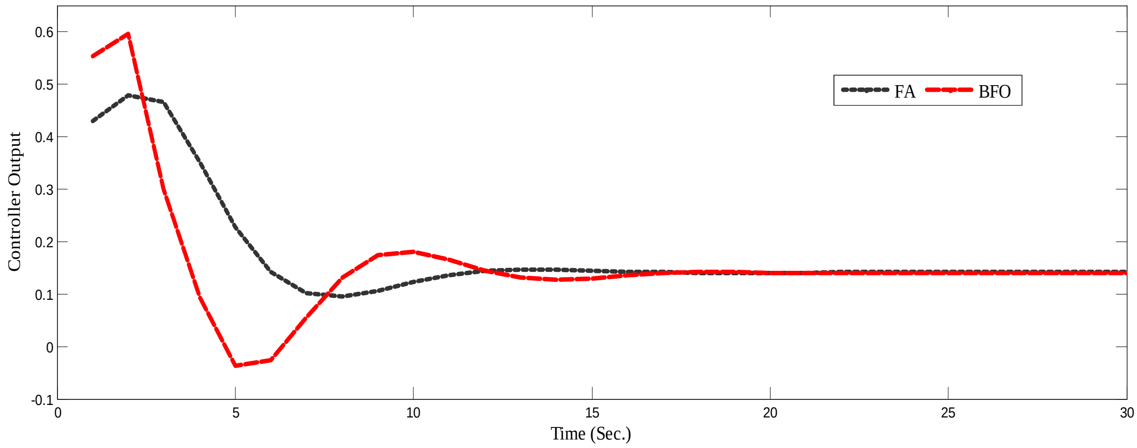

Fig. 9: Controller output for servo Response of different PID Controllers Tuning

The performances of the FA and BFO tuned PID controllers are tested on the FOPTD model of the neutralization process using equation (9). The servo control and regulatory control tests are also conducted. The servo response of PID Controller tuning using both the algorithms is shown in Fig.8 and its corresponding controller output is shown in Fig.9. It is observed that the performance of both the algorithms is very good and also their error minimization criterion and Time Domain Specification results are very closer. However in deep analysis it is found that the performance of FA has better set point tracking than BFO. Also, number of iteration indicates that the Firefly algorithm has excellent convergence speed ahead of BFO.

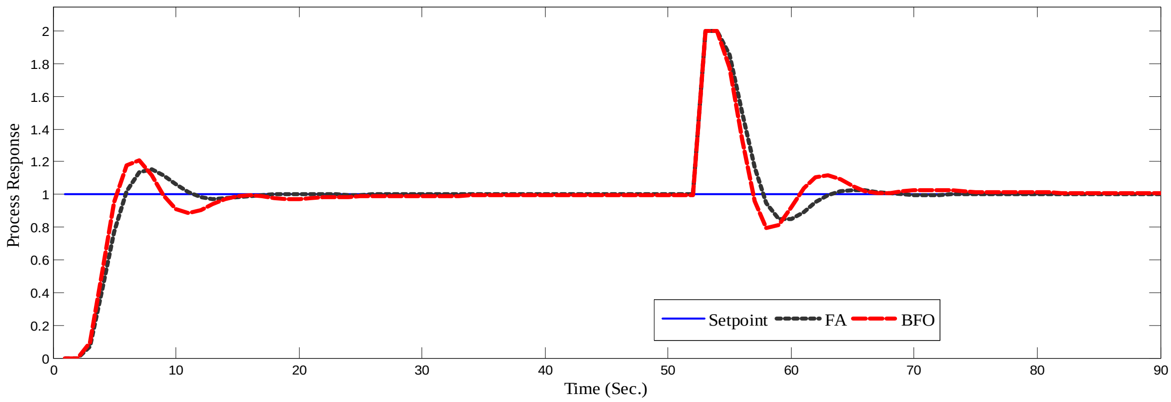

Fig. 10: Regulatory Response for different PID Controllers Tuning

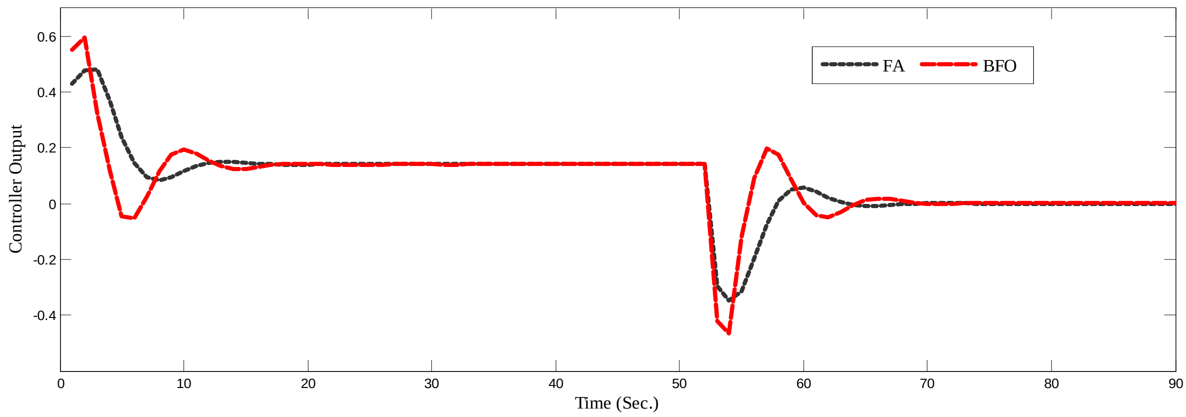

Fig. 11: Controller output for Regulatory Response of different PID Controllers Tuning

In continuation of the servo response analysis, regulatory response analysis is also executed to study the fitness of the controller to evaluate the performance of FA and BFO algorithms. In this test, a load disturbance is given after 50 seconds to study the load disturbance capability of the controller. The Fig.10 and Fig.11 show that are shows the Regulatory Response and corresponding controller output. It indicates that the firefly algorithm based controller has quickly settled out after the disturbance compared with BFO algorithm based controller. In this study the both the servo response and regulatory response indicates that Firefly algorithm based controller has marginally better performance over the BFO algorithm based controller.

CONCLUSION

The performance of Firefly Algorithm (FA) and Bacterial Foraging Algorithm (BFO) is demonstrated with help of mathematical model of pH neutralization process. The test result shows that the computational speed of convergence of FA is faster than convergence of BFO. In the each algorithm 5 trials are carried out to obtain the best optimized controller parameters. Among the 5 trial values, the best convergence value from each algorithm is selected for the study. The servo response and regulatory response test are conducted to study the performance of both the algorithm based controllers. The comparative study is carried out with help of time domain specifications and error minimization. The study tested that the performance of Firefly algorithm based PID controller which produced marginally better result compared with the Bacterial Foraging Algorithm for the non linear pH neutralization process.

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